When Do Introspection Axioms Matter for Multi-Agent Epistemic Reasoning?

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- Do we really need to introspect and
- if we don’t even try to introspect, do introspection axioms still matter?

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- Provide some details.
- Discuss previous works and possible extensions.

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Formal 2 In the parsing tree restricted to the modal operators, every path from the roots to the leaves is agent-alternating. Modalities of the same agent are never adjacent.

This idea is not new. We'll say more about previous works later.

Agent-alternating formulas

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Define a family $\{\mathcal{L}_{-a}\}_{a\in A}$ of languages through the following simultaneous induction:

$$\mathcal{L}_{-a} \ni \varphi ::= p \mid \Box_x \psi \mid \neg\varphi \mid (\varphi \wedge \varphi)$$

where $p \in \mathrm{Prop}$ and $x \in A \setminus \{a\}$ while $\psi \in \mathcal{L}_{-x}$.

Then the language $\mathcal{L}_{alt}$ is defined inductively by

$$\mathcal{L}_{alt} \ni \varphi ::= p \mid \chi \mid \neg\varphi \mid (\varphi \wedge \varphi)$$

where $p \in \mathrm{Prop}$ and $\chi \in \bigcup_{a\in A} \mathcal{L}_{-a}$.

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- Formulas that don't start with $\Box_a$ have been called "objective formulas for a". $\mathcal{L}_{-a}$ is its hereditary version, also studied in the same line of research.

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$\Box_a(p \wedge \Box_b \Box_a q)$ is agent-alternating. $\Box_a(\Box_b \Box_a p \wedge \Box_a q)$ is not.

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- With or without the introspection axioms, you can make the same inference moves.
- I.e., the logic (what follows from what) doesn't change.
- This means we can do formalization and reasoning with certainty in certain cases while being undertain about what $\Box$ really means and which logic it really follows in full generality.

Question formalized

Hence, the question "if we don't even try to introspect, do introspection axioms still matter?" is formalized as follows.

Main question

For which modal logic L and which axiom φ,
$\mathsf{L} \cap \mathcal{L}_{alt} = \mathsf{L}\varphi \cap \mathcal{L}_{alt}$?

More generaly, for which modal logics L and L′,
$\mathsf{L} \cap \mathcal{L}_{alt} = \mathsf{L}' \cap \mathsf{L}_{alt}$?

We have a language $\mathcal{L}_{alt}$, and we ask its power to collapse logics.

Question answered

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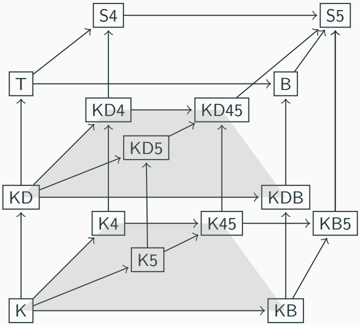

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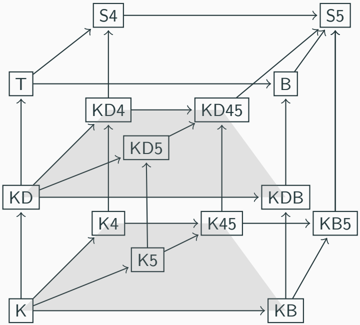

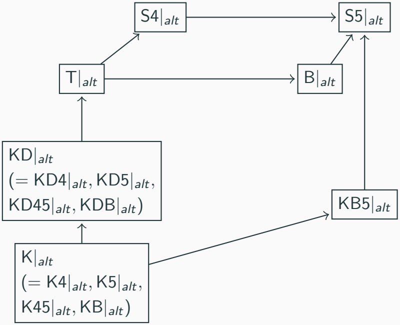

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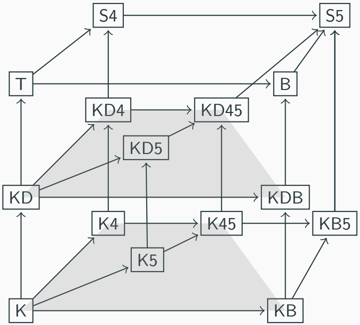

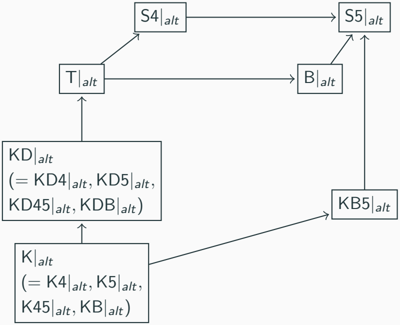

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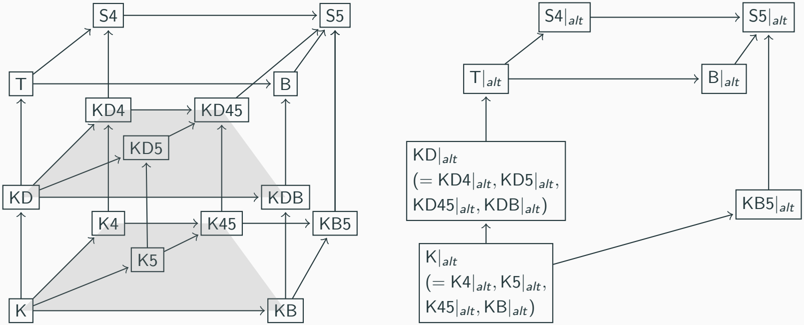

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- KB5 almost has T, so no collapse.

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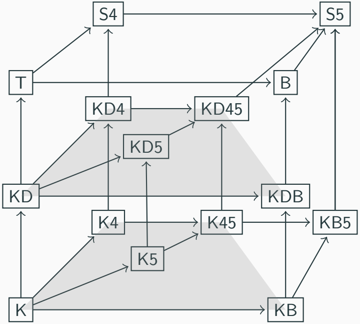

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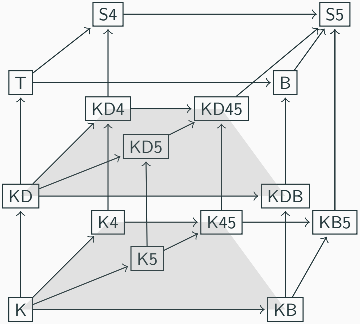

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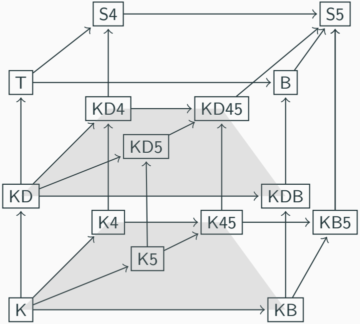

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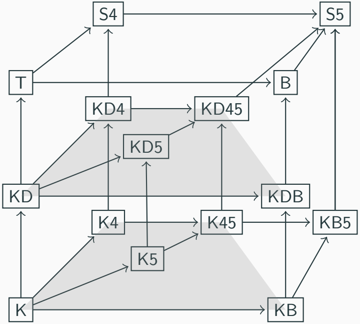

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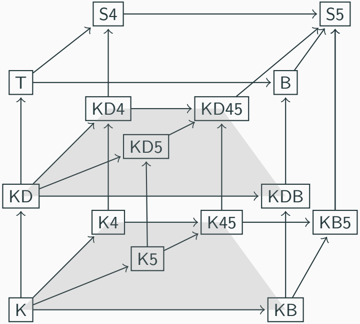

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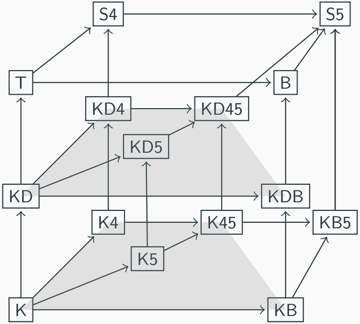

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- For KB5, we add $\Diamond_a \Diamond_b \top$ in the antecedent since in KB5 this gaurantees that $\Box_b$ is factive.

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- Once unravelled agent-alternatingly, we can add arrows and still be agent-alternatingly bisimilar.

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  - And $\mathcal{L}_{alt}$ is not collapsing 45 trivially. It says all that can be said (in $\mathcal{L}_{alt}$) among T and E models.
- We also showed that 4 and 5 are necessary among the logics in the Cube for $\mathcal{L}$ and $\mathcal{L}|_{alt}$ to be equi-expressive.

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- The idea of agent-alternating is very prominent in Bernheim's one of the first papers defining rationalizable strategies, resulting in an agent-alternating system of beliefs.
  - In fact, we can formalize and prove using Kripke models that agent-alternating common belief of rationality implies the played strategy is rationalizable.

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- We can add the usual common knowledge operator and see if there's a natural fragment in line with the idea of "agent alternating" that collapse logics.
- We only have non-collapsing results now. The usual common knowledge is by itself not agent-alternating...

$$\left(\bigwedge_{b \in A \setminus \{a\}} (\Box_b p \wedge C\Box_b p \wedge \Box_b \Box_a p \wedge C\Box_b\Box_a p) \wedge \Box_a p \right) \to Cp.$$

is valid with transitivity, but not otherwise. There should be collapse with infinitely many agents.

Future work

- Collapsing results for natural fragments with the standard common knowledge.

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- Finally, can we say more about the practical sufficiency of $\mathcal{L}_{alt}$?

Thank you!